2025.M29

2025L003A1EL

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2025

Mathematics

Paper 1

Higher Level

Friday 6 June   Afternoon 2:00 - 4:30

300 marks

Examination Number

Date of Birth

For example, 3rd February 2005 is entered as 03 02 05

Centre Stamp

The 2025 examination papers were adjusted to compensate for disruptions to learning due to COVID-19. This examination paper does not necessarily reflect the same structure and format as the examination papers of past or subsequent years.

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Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	4 questions

Answer questions as follows:

- **any five** questions from Section A – Concepts and Skills
- **any three** questions from Section B – Contexts and Applications.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet. If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

In general, diagrams are not to scale.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A Concepts and Skills 150 marks

Answer **any five questions** from this section.

Question 1 **(30 marks)**

(a) Solve the following inequality for $x \in \mathbb{R}$:

$$|x - 3| \leq 12$$

(b) Multiply out and simplify:

$$\left(4x - 10\sqrt{x}\right)\ (2x + 5\sqrt{x} - 7)$$

(c) $(2x + 3)$ is a factor of $4x^3 - 12x^2 - 7x + 30$.
Use this information to find the **three** solutions to the following equation in x:

$$4x^3 - 12x^2 - 7x + 30 = 0$$

$x =$ ______________ , $x =$ ______________ , and $x =$ ______________

Question 2 **(30 marks)**

(a) A function is defined for $x \in \mathbb{R}$ by:

$$f(x) = 6 + x^2 + \sin 4x$$

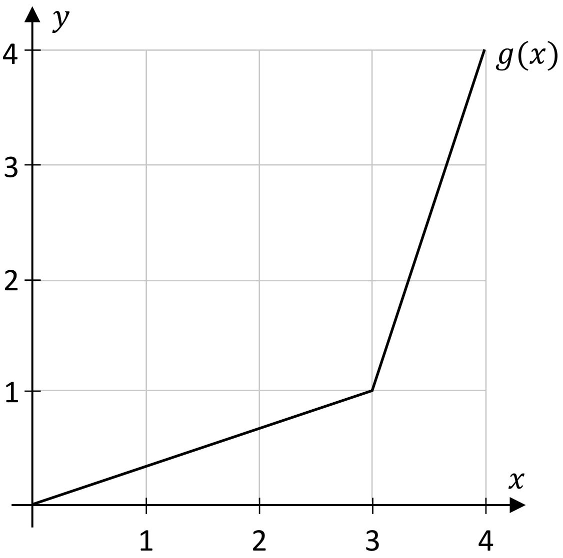
(i) Find $f'(x)$, the derivative of f with respect to x.

(ii) Find the **equation** of the tangent to the curve $y = f(x)$ at the point where $x = 0$.
Give your answer in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{Z}$.

(b) The function $g(x)$ is defined for $0 \leq x \leq 4$, $x \in \mathbb{R}$.

Its graph is shown in the diagram on the right, and is made up of two line segments.

Use the graph of $g(x)$ to answer parts **(b)(i)**, **(b)(ii)**, and **b(iii)**.



(i) State the range of values of x for which $g'(x) > 2$.

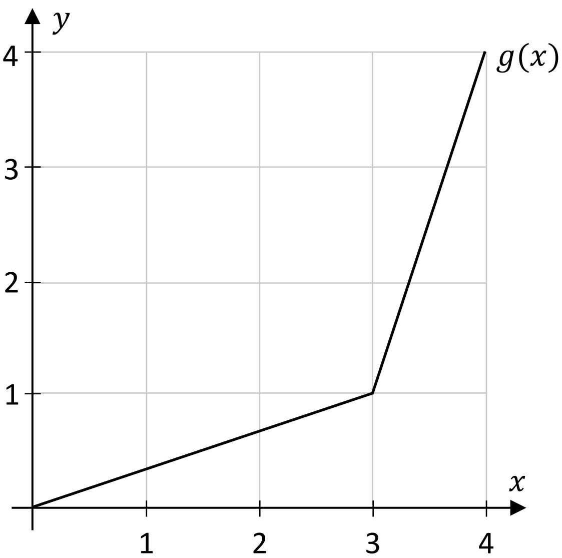
(ii) Find the value of $g(g(3))$. Give your answer in the form $\frac{a}{b}$ where $a, b \in \mathbb{N}$.

Show your work on the graph.

(iii) The graph of $y = g(x)$ is shown again on the diagram below.

Draw and **label** the graph of $y = g^{-1}(x)$ on the same diagram, for $0 \leq x \leq 4$, $x \in \mathbb{R}$, where g^{-1} is the inverse of the function g.

Hint: the graph of g^{-1} is the image of the graph of g under axial symmetry in the line $y = x$.



Question 3 **(30 marks)**

(a) The function $f(x)$ is defined for $x \in \mathbb{R}$ by:

$$f(x) = (3x^5 - 4)^{28}$$

Find an expression for $f'(x)$. You do **not** need to simplify your answer.

(b) The function $g(x)$ is defined for $x \in \mathbb{R}$, $x \neq 3 \cdot 5$, by:

$$g(x) = \frac{3}{2x - 7}$$

By finding $g'(x)$, show that this function has **no** local maximum or minimum points.

(c) $k \in \mathbb{R}$ is a constant, and:

$$\int_0^k e^{5x} dx = 9$$

Use this to find the value of k.

Write your answer in the form $k = \frac{\ln a}{b}$, where $a, b \in \mathbb{N}$.

Question 4 **(30 marks)**

In this question, $i^2 = -1$.

(a) Write the complex number $\frac{2+3i}{4-5i}$ in the form $a + bi$, where $a, b \in \mathbb{R}$.

(b) Use **de Moivre's theorem** and the expression $(\cos\theta + i\sin\theta)^2$ to prove that the following is true, for any angle θ:

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

(c) Use **de Moivre's theorem** to find **two** values of z for which:

$$z^6 = -64\,i$$

Give each answer in the form $c + di$, where $c, d \in \mathbb{R}$. Leave c and d in surd form.

$z =$ ____________________________________

and $z =$ ____________________________________

Question 5 **(30 marks)**

(a) The function $g(x)$ is defined for $x \in \mathbb{R}$ by:

$$g(x) = 5x^2 + 20x - 12$$

Write $g(x)$ in the following form, where $a,\ h,\ k \in \mathbb{Z}$ are constants:

$$g(x) = a(x + h)^2 + k$$

$g(x) =$ ____________________________________

(b) p is a positive constant.
Use the laws of logs to write the expression:

$$\ln[\,(e^3\,p)^5\,]$$

in the form $c + d\ln p$, where $c,\ d \in \mathbb{Z}$ are constants.

(c) Below is a pair of simultaneous equations in x and y, where $n \in \mathbb{R}$ is a constant.

One of the solutions to this pair of equations is on the y-axis.
Use this information to find the value of n.

$$2x - y = 7$$
$$x^2 + y + 2y^2 = n$$

$n =$ ____________

Question 6 **(30 marks)**

(a) Write down, in descending powers of p, the first 3 terms in the binomial expansion of:

$$(2p+3)^7$$

Give each term in its simplest form.
For example, the first term should be of the form ap^7, where a is a constant.

(b) $h(x)$ is the following function of x, where m and r are positive constants and $x \in \mathbb{R}$:

$$h(x) = 6m\,x^2 - 4r\,x + 54m$$

(i) The equation $h(x) = 0$ has **exactly one** solution.
Use this to show that $r = 9m$.

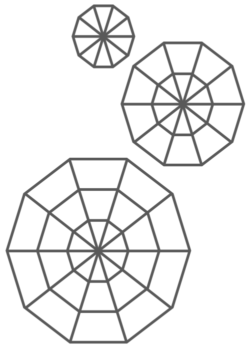
(ii) Use the fact that $r = 9m$ to find the value of this solution.

$x =$ ____________

Section B	Contexts and Applications	150 marks

Answer **any three questions** from this section.

Question 7 **(50 marks)**



Spiders build webs out of silk.
This question involves two different models of how a spider might build its web. The functions in each model give lengths in centimetres.

(a) In the first model, the web is made in stages.
First, the spider makes Stage 1 of the web.

It then adds **extra** silk to Stage 1 to make Stage 2 of the web, and so on.

The table below shows the values of $S(n)$, the **total** length of silk in each stage, in the first three stages of this model.

The table also shows $A(n)$. For $n > 1$, $A(n)$ is the length of **extra** silk needed to make Stage n from Stage $n - 1$. $A(1)$ is set to be $15{\cdot}8$.

Stage	**1**	**2**	**3**
$\boldsymbol{S(n)}$**: total length of silk**	$15{\cdot}8$	$37{\cdot}8$	66
$\boldsymbol{A(n)}$**: extra length of silk**	$15{\cdot}8$		

(i) Use the values in the table to show that $A(2) = 22$ **and** find the value of $A(3)$.
Write the values of $A(2)$ and $A(3)$ into the table above.

Show that $A(2) = 22$:	Find the value of $A(3)$:

The numbers $A(1)$, $A(2)$, $A(3)$, ... form an **arithmetic** sequence.

(ii) Find an expression in n for $A(n)$, where $n \in \mathbb{N}$.

(iii) Hence, or otherwise, find the value of $A(100)$.

(iv) $S(n)$ is the **total** length of silk needed to make Stage n of the web, where $n \in \mathbb{N}$.
By using the formula for the sum of an arithmetic series, or otherwise, show that:

$$S(n) = 3{\cdot}1n^2 + 12{\cdot}7n$$

(v) Stage k is the first stage for which the **total** length of silk is more than **10 m**, in this model. Using the expression for $S(n)$ above, solve an equation to find the value of k.

Remember that $S(n)$ gives the total length of silk, in **centimetres**.

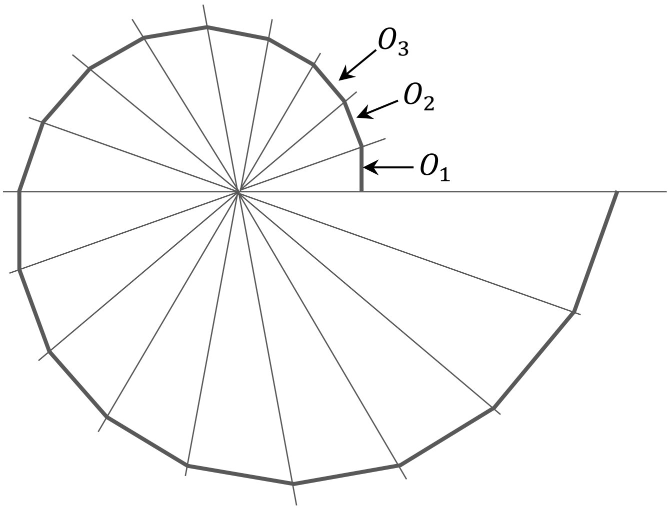
This question continues on the next page.

(b) In the second model, the web is made in laps. The diagram on the right shows the first lap of the web.

The line segments marked O_1 , O_2 , and O_3 in the diagram are the first 3 **orbitals** of the web.

The lengths of the orbitals of the web form a **geometric** sequence.

The length of O_1 is $0{\cdot}5$ cm and the length of O_2 is $0{\cdot}53$ cm.



(i) Find the length of O_3.

(ii) Write an expression in n for the **total** length of the first n orbitals of the web, where $n \in \mathbb{N}$.

(iii) There are exactly 18 orbitals in each **lap** of the web.
Write an expression in k for the **total** length of the **first *k* laps**, where $k \in \mathbb{N}$.

Question 8 **(50 marks)**

(a) Jacob buys a new kayak in a shop. The kayak is usually priced at €870.
This price is reduced by 15% in a sale.

Jacob gets a further reduction of 10% on this reduced price, because he is a member of the shop's loyalty club.

Find the price that Jacob pays for the kayak.

(b) Jacob buys a paddle online. The paddle costs \$$95$.

Jacob thinks that the exchange rate being used is €$1 = \$1{\cdot}183$.
The actual exchange rate being used is €$1 = \$\, d$, where $d \in \mathbb{R}$.

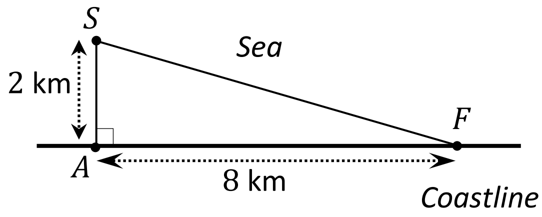
As a result, the paddle costs €$1{\cdot}02$ more than Jacob thought that it would.

Use this to find the value of d, correct to 3 decimal places.

This question continues on the next page.

Jacob takes part in a race, which involves kayaking and running. He must get from the point S in the sea to the point F on the coastline.

A is another point on the coastline.
$|SA| = 2$ km, $|AF| = 8$ km, and
$\angle SAF$ is a right angle.



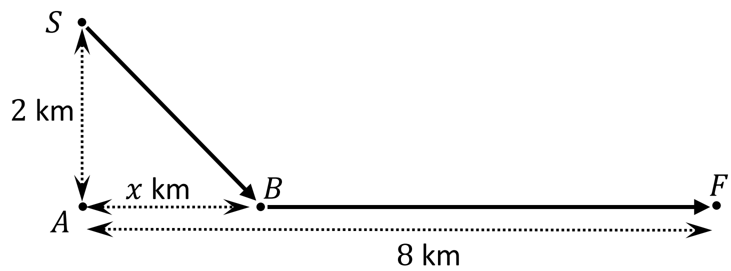
Jacob kayaks at an average speed of 6 km/hour and runs at an average speed of 12 km/hour.

(c) Find how long it would take Jacob in **total** to kayak from S to A, and then run from A to F.

(d) Find how long it would take Jacob to kayak directly from S to F, correct to the nearest minute.

The point B is on $[AF]$,
and $|AB| = x$ km,
where $0 \leq x \leq 8$, $x \in \mathbb{R}$.

(e) Jacob is going to kayak from S to B, and then run from B to F.



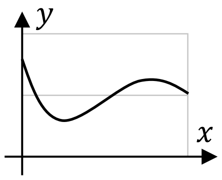
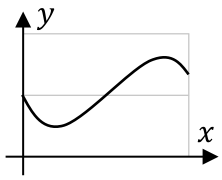
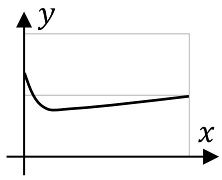
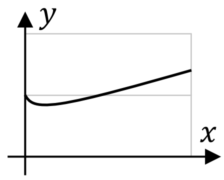
(i) Find an expression in x for T, the total time, in hours, that it will take Jacob to get from S to B to F.

(ii) Jacob knows that there is one value of x for which $T(x)$ is a minimum ($0 \leq x \leq 8$).
Solve the following equation to find this value of x, correct to 3 decimal places:

$$T'(x) \;=\; \frac{x}{6\sqrt{x^2+4}} - \frac{1}{12} = 0$$

(iii) There are no other values of x in $[0, 8]$ for which $T'(x) = 0$, apart from the answer to part **(e)(ii)**.

Use this fact, and your answers from parts **(c)** and **(d)**, to find which **one** of the following graphs could represent $T(x)$, for $x \in [0, 8]$. Justify your answer fully.

			
Graph A	**Graph B**	**Graph C**	**Graph D**

Answer (**A**, **B**, **C**, or **D**):

Justification:

Question 9 **(50 marks)**

Dani drives a car.

(a) The fuel consumption, F, of Dani's car depends on the speed of the car, c.
For one particular journey, F is given by:

$$F(c) = 0{\cdot}05\, c^2 - 8{\cdot}5\, c + 800$$

where F is in litres per $10\ 000$ km, and c is in km/hour, with $40 \leq c \leq 120$.

(i) Show that there is no difference between the fuel consumption (F) when Dani's car is travelling at 60 km/hour and when it is travelling at 110 km/hour.

(ii) Find an expression for $\frac{dF}{dc}$, the rate of change of fuel consumption with respect to speed.

During part of the journey, the speed, c, of Dani's car at time t is given by:

$$c = 78 + 9\ln(t^2)$$

where t is the time in minutes, $1 \leq t \leq 10$, and c is in km/hour.

(iii) Use this, and your answer to part **(a)(ii)**, to find the value of $\frac{dF}{dc}$ when $t = 7$.
Give your answer correct to 1 decimal place.

(iv) Show that the rate of change of the car's speed with respect to time is given by:

$$\frac{dc}{dt} = \frac{18}{t}$$

(v) Use your answers to parts **(a)(iii)** and **(a)(iv)** to find the rate of change of the car's fuel consumption, F, with respect to time, at the instant when $t = 7$ minutes.

Give your answer in (litres per $10\ 000$ km) per minute.

This question continues on the next page.

(b) Over the first 8 seconds that Dani is driving her car, the car's speed, in km/hour, can be approximated using the following function $v(t)$:

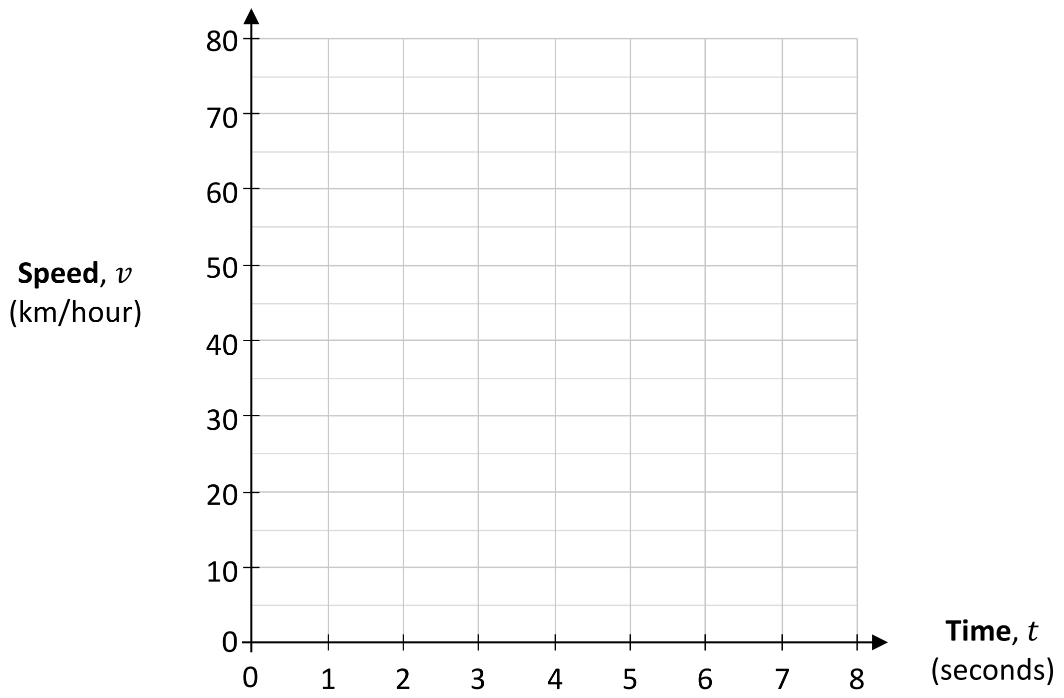
$$v(t) = \begin{cases} 8e^{0{\cdot}4t} - 8\,, & 0 \leq t \leq 4 \\ -t^2 + 24t - 48{\cdot}4\,, & 4 < t \leq 8 \end{cases}$$

where $t \in \mathbb{R}$ is the time, in seconds, after the car starts moving.

(i) Fill in the table below to show the values of $v(t)$ for the given values of t, up to $t = 8$. Give each value correct to 1 decimal place, where appropriate.

Time, t (seconds)	0	1	2	3	4	5	6	7	8
Speed, v (km/hour)	0		9·8		31·6			70·6	79·6

(ii) Hence, draw the graph of the function $y = v(t)$ on the axes below, for $0 \leq t \leq 8$, $t \in \mathbb{R}$.



(iii) Use integration, and $v(t) = -t^2 + 24t - 48{\cdot}4$, to find the average speed of Dani's car for $4 < t \leq 8$.

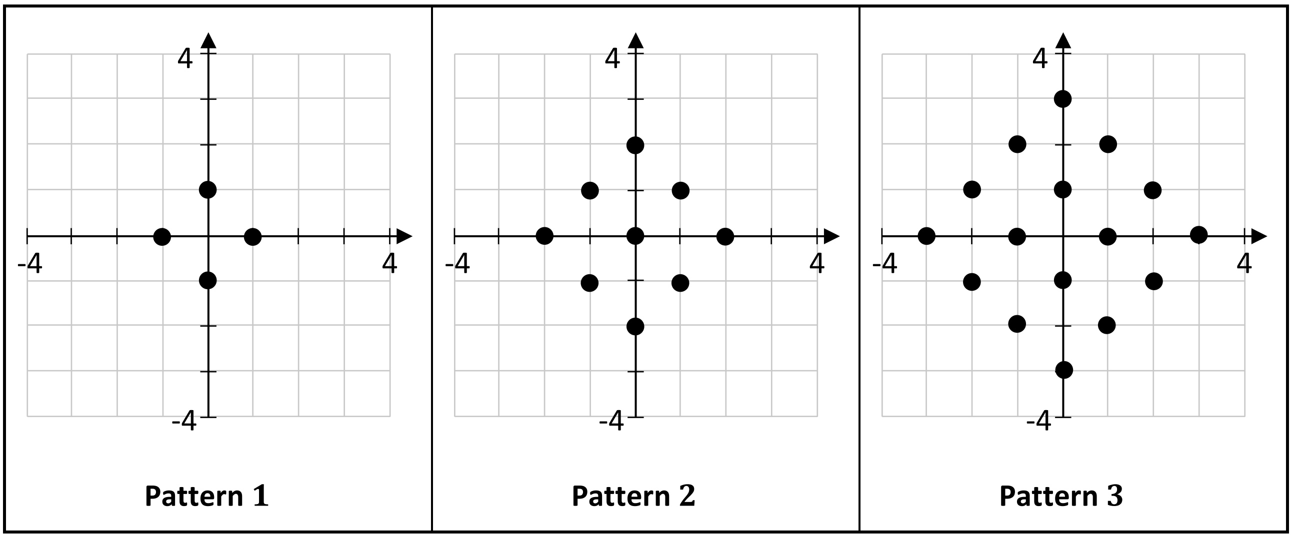
Give your answer in km/hour, correct to 1 decimal place.

Question 10 **(50 marks)**

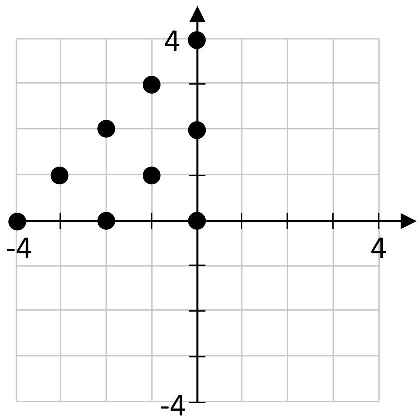
The first three patterns in a sequence of patterns are shown below. Each pattern is made up of dots (●) at points in the co-ordinate plane that have **integer co-ordinates**.

As shown below, Pattern 1 has a dot at all such points that are a distance of 1 unit from $(0, 0)$, and Pattern 2 has a dot at all such points that are a distance of 1 unit from a point in Pattern 1.

As the sequence continues, Pattern $n + 1$ has a dot at all such points that are a distance of 1 unit from a point in the previous pattern (Pattern n). This is true for all $n \in \mathbb{N}$.



(a) 9 of the dots in Pattern 4 are shown on the co-ordinate diagram below.
Draw in the missing dots to complete Pattern 4.
Do **not** mark any points that are not in Pattern 4.



(b) In Pattern 2000, there are 4 points that are a distance of 2000 units from $(0, 0)$.
Write down the co-ordinates of these 4 points.

(c) What is the smallest value of $n \in \mathbb{N}$ for which the point $(4, 4)$ is in Pattern n?
Do **not** draw on your diagram from part **(a)**.

This question continues on the next page.

$Q(n)$ is the proportion of points with integer co-ordinates (a, b), where $|a|, |b| \leq n$, that are in Pattern n of the sequence.

(d) **(i)** Rearrange the formula $t = 2n + 1$ to write n in terms of t.

$n =$ __________________

$Q(n)$ is given by the following formula, for any $n \in \mathbb{N}$:

$$Q(n) = \frac{(n+1)^2}{(2n+1)^2}$$

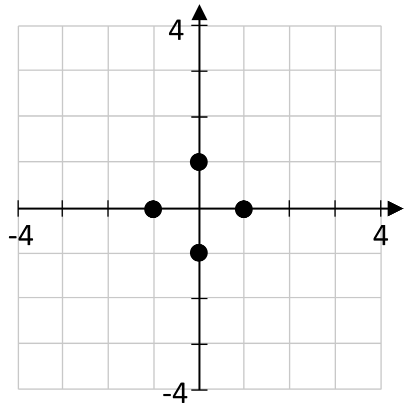
(ii) By **substituting your answer** from **(d)(i)** into the expression for $Q(n)$ above, show that:

$$Q(n) = \frac{t^2 + 2t + 1}{4t^2}$$

(iii) Find the value of Q_{∞} which is given by:

$$Q_{\infty} = \lim_{t \to \infty} \frac{t^2 + 2t + 1}{4t^2}$$

(e) $H(n)$ is the **total number of dots** in Pattern n of the sequence, for $n \in \mathbb{N}$.



Pattern 1

(i) Write down the value of $H(1)$.

$H(1) =$ ☐

When n is a natural number, $H(n+1)$ can always be found from $H(n)$, using this formula:

$$H(n+1) = H(n) + 2n + 3$$

(ii) Using this fact, prove the identity on the grid below.
Use proof by **induction**.

Prove that: $H(n) = (n+1)^2$ for all values of $n \in \mathbb{N}$

You may use this page for extra work.
Label any extra work clearly with the question number and part.

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Label any extra work clearly with the question number and part.

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Leaving Certificate – Higher Level

Mathematics Paper 1

Friday 6 June

Afternoon 2:00 - 4:30